

(3 Hours)

[Total Marks: 80]

Note:

- Question No. 1 is compulsory.
- Answer any three from the remaining five questions.
- Assume suitable data if necessary and justify the same.
- Figures to the right indicate the marks.

1 Each question carry five marks

20

- Define and compare absolute error and relative error in numerical computation with one example each.
- Explain how golden ratio can be used to calculate the new search range to solve one dimensional optimization problem using Golden section search?
- Explain why bracketing methods always converge whereas open methods may sometimes diverge?
- Define unbounded solution in LPP. Explain how this can be identified while solving the problem using graphical method with an example.

2 a What is meant by interpolation? What are the steps of Newton's divided difference interpolation method and discuss the error in this interpolation. How the accuracy can be improved in this interpolation? 10

b Using Newton's Divided difference method of order 4 and the given set of X and Y, find the interpolated value of Y at given $X=3.20$ with maximum accuracy. 10

X	1	2	3	4	5	6
Y	8.0	11.0	16.0	23.0	32.0	43.0

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- 3 a What do you understand by one-dimensional unconstrained optimization? Write the algorithm of Quadratic interpolation method to find the maximum of a function. Employ Quadratic interpolation method to find the maximum of $f(x) = -x^2 + 8x - 12$ on initial guesses $x_0 = 0$, $x_1 = 2$, $x_2 = 6$. Perform only two iterations. 10
- b State the necessary and sufficient condition for solving multi variable optimization problem with equality constraint using the method of Lagrange Multipliers. 10
- Using Lagrange's multiplier method,
Maximize $Z = 7x_1 - 0.3x_1^2 + 8x_2 - 0.4x_2^2$; subject to $4x_1 + 5x_2 = 100$ and $x_1, x_2 \geq 0$.
- 4 a Give the algorithm of Secant method to find the root of an equation. Explain geometrically how the choice of guess for new interval and new root are made in any iteration of Secant method. 10
- b An LC circuit obeys following law $L \frac{d^2 i}{dt^2} + \frac{1}{C} i = E_0 \sin t$ with the initial conditions $i = 0$, and $\frac{di}{dt} = 6$, when $t = 0$. For the given equation $L = 0.5H$, $C = 0.2F$ and $E_0 = 10V$ then find i after 0.1 sec taking $h = 0.1$ using Runge Kutta fourth order method. 10
- 5 a Solve the following LP problem using simplex method. 10
- Max $Z = 5x_1 + 3x_2$
subject to $x_1 + x_2 \leq 2$; $5x_1 + 2x_2 \leq 10$; $3x_1 + 8x_2 \leq 12$; and $x_1, x_2 \geq 0$
- b What are single step and multi-step methods of solving first order ordinary differential equation? Distinguish between them. 05

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- c Using Picard's method find the solution of $\frac{dy}{dx} = x^2 + y^2$ at $x=0.2$ with the initial condition as $y(0) = 2$. 05
- 6 a A furniture manufacturer makes two products: chairs and tables. 10
Processing of these products is done on two machines A and B. A chair requires 2 hours on machine A and 6 hours on machine B. A table requires 5 hours on machine A and no time on machine B. There are 16 hours per day available on machine A and 30 hours on machine B. Profit gained by the manufacturer from a chair and a table is Rs. 2 and Rs. 10 respectively. Solve this problem using graphical method to find the daily production of each of the two products.
- b Find a polynomial for $f(x)$, where $f(0)=1$, $f(1)=2$ and $f(3)=5$ using Lagrange's method. 05
- c Explain the advantages of pivoting for solving Linear algebraic equations using LU decomposition. 05